Centre de Recherche en Economie Publique et de la Population

CREPP WP No 2009/08

Unemployment and Retirement in a Model with age-specific Heterogeneity

Mathieu Lefebvre

December 2009

CREPP - Centre de recherche en Economie Publique et de la Population
Boulevard du Rectorat 7 (B31)
4000 Liège Belgium

CREPP Working PaPer Series

Unemployment and retirement in a model with age-specific heterogeneity

Mathieu Lefèbvre
CREPP, University of Liège
Boulevard du Rectorat 7 (B31)
4000 Liège Belgium

Abstract

This paper presents a model where young and old workers compete for one type of jobs in the presence of retirement opportunity. Within this framework, we show that increased retirement opportunities (such as early retirement) has most of the time a depressing impact on the unemployment rate. Indeed the number of vacancies posted by firms is influenced by the probability that an old worker is going into retirement. We show that the degree to which younger workers are influenced by retirement of older workers depends on the relative productivity of young and older workers. It is only when older workers are much more productive than young workers that retirement may benefit to unemployment.

1 Introduction

In the last few decades, many countries have experienced a major shift in the labor force participation of older workers. The development of social security programs has had a significant impact on labor market behavior. As documented in Gruber and Wise (1999), many countries have introduced a variety of early retirement schemes and long-term unemployment insurance programs that allow for older workers to retire well before the normal retirement age. Consequently, people are retiring earlier and the labor force participation of older workers is falling.

In public discussions, the idea of retirement as an instrument for reducing unemployment emerged early on. Most of the programs, inducing early retirement, have been introduced directly to combat unemployment problems facing younger workers. The argument is based on the idea of work sharing or the assumption of a lump of labor. Kapteyn et al. (2004) explained that the idea relies on the simple notion that, in a given period, a fixed amount of labor input required to produce a fixed volume of goods and services can be shared between persons who are already employed and those who are unemployed. It is usually argued that a trade-off can be made between the positively valued leisure of the older employed and the unwanted leisure of the younger unemployed.

Few studies have addressed the validity of such a claim. Boldrin et al. (1999) looked at the labor force participation rate of older workers and the unemployment rate of younger cohorts in European countries. Their estimates do not show evidence of a negative correlation between the exit rates from the labor force of older workers and the variation in the unemployment rates among young workers. There are several reasons for the failure of such a policy. One simple reason may be that the central assumption is wrong, i.e. workers are not perfectly substitutable. Surveys of empirical literature on substitution patterns for workers of different ages show no homogeneous evidence. Hammermesh and Grant (1979) and Hammermesh (1987) provide a critical survey of the large number of studies at hand and the results are quite heterogeneous. Their conclusion is that the young worker's own-wage elasticity of demand exceeds unity, but the degree to which young and older workers are substitutes for each other is unclear. Hebbink (1993) finds that the demand for young and older workers changes in the same direction if one of their wages changes. Young and older workers are therefore complementary factors of production.

In this paper, we intend to provide an analysis of age-related effects in the labor market when older workers are affected by retirement programs. We develop a model of equilibrium unemployment following the Diamond-Mortensen-Pissarides model, in which we consider two types of worker, young and older, who are different in individual attributes but who compete for the same job. Previous studies have focused on different age-groups in the labor market (see Shimer (2001) and Hairault et al. (2006) for example), especially on the impact of the aging process on the labor outcomes of different age groups. However, none were concerned with the impact on younger workers of the retirement behavior of older people. In this respect, we deviate from previous studies by incorporating productivity differentials and differences in age-specific separation risks due partly to the older workers' opportunity to retire. We also model the life-cycle behavior of the workers.

Within this framework, we show that a higher propensity to retire has mainly negative externalities on labor outcomes, especially in the case of younger cohorts. First, when workers differ with respect to the probability of ending a match with an employer, with some workers having a higher probability of quitting, firms will anticipate this and restore profitability by opening up fewer vacancies in equilibrium. Second, when "high exit" workers are much more productive than the other workers, this may generate a positive externality. The supply of vacancies will depend positively on the fraction of more productive workers searching for a job.

Under certain conditions, this second externality dominates the first one. Both effects are common knowledge in the literature and this has been highlighted by Ortega (2000) paper on migration regarding foreign workers facing higher search costs and therefore receiving lower wages and also in Gautier (2002) regarding low-skilled and high-skilled workers competing for low-skilled jobs. However, in the context of retirement opportunities, the probability of exit also applies to unemployed people and thus directly affects the number of older job-seekers. As a consequence, it reduces the incentive for firms to open up more vacancies in order to attract older workers. We show that an increase in levels of retirement has the effect of increasing the level of younger workers' unemployment. Even if older workers are sufficiently productive to induce the opening up of more vacancies, the fall in the number of older job-seekers reduces the positive effect. Finally, we identify a high level of productivity differential above which an increase in retirement will benefit all workers. Unfortunately such a level is very unlikely in reality.

The plan for the rest of the paper is as follows. In Section 2, we outline the equilibrium unemployment model with age-related heterogeneity in the labor force and early retirement possibility. Section 3 presents some simulation results based on a calibrated version of the model. Section 4 proposes an econometric framework to assess the impact of early retirement on unemployment. Section 5 concludes.

2 The model

2.1 Environment

The economy is populated by a continuum of firms and workers. Firms produce a unique final good and employ only one worker. Workers may be young or old (indexed y, o) and the share of young individuals in the population is given by p. The exogenous probability of becoming old for a young worker is given by η_y and older workers go to retirement with probability η_o. Thus workers remain young for a period equal to $1/\eta_y$ and they are old during a period equal to $1/\eta_o$. We assume that the risk to becoming a retiree is supported during a short duration time so that $\eta_o > \eta_y$. The aggregate workforce is assumed to be constant and normalized to 1.

A job can be filled by either type of worker but the productivity of the job depends on the type of worker employed. A job occupied by a young worker produces γ while a job occupied by an older worker produces $\delta\gamma$. At this point, there is no *a priori* reason to assume δ to be larger or smaller than 1. In this respect, we introduce heterogeneity in the labor force in a way similar to Acemoglu (1997) and Gautier (2002), who distinguished between high-skilled and low-skilled workers. However when hiring a younger worker the firm interiorizes that the worker will become old with a probability η_y.

Workers and vacancies meet according to a CRS matching function which is increasing in the relevant amount of searchers and vacancies and is concave, $M = M(v, u)$ with v denoting the number of vacancies and u being the number of young and old unemployed. Vacancies are equally open to young and older workers but search frictions limit the matching of unemployed workers and vacancies. Another modeling strategy would be to allow firms to aim their vacancies at a specific type of worker. In this case we should consider two distinct matching functions according to the age of the workers. This would not change the following analysis and our main results would hold except for the trading externalities when both types of worker are competing for the same jobs.Vacancies are equally open to young and older workers but search frictions limit the matching of unemployed workers and vacancies. Another modeling strategy would be to allow firms to aim their vacancies at a specific type of worker. In this case, we would consider two distinct matching functions according to the age of the workers. This would not change the following analysis and our main results would hold except for the trading externalities when both types of worker are competing for the same jobs. Diamond (1982) showed that parties on the same side of the market create in general negative search externalities, while parties on different sides of the market create positive search externalities.

Given the CRS property of the technology and the standard matching assumption, the probabilities of a firm filling a vacancy is given by $q(\theta) \equiv \frac{m}{v} = m\left(1, \frac{1}{\theta}\right)$ and equivalently the probability of a worker contacting a firm is represented respectively by $\frac{m}{u} = m(\theta, 1) = \theta q(\theta)$. The tightness of the labor market is given by $\theta = v/u$. Under the CRS assumption the probability of filling a job is the sum of the probability of hiring a young worker and the probability of hiring an old worker:

$$q_y(\theta) = \frac{pu_y}{u} q(\theta) = p\pi q(\theta) \tag{1}$$

$$q_o(\theta) = \frac{(1-p)u_o}{u} q(\theta) = (1 - p\pi) q(\theta) \tag{2}$$

Note that, given the properties of the matching technology, $q_y(\theta)$ and $q_o(\theta)$ are decreasing in vacancies and increasing in the number of job-seekers: $q_y^{'} \leq 0$ and $q_o^{'} \leq 0$, while $\theta q_y(\theta)$ and $\theta q_o(\theta)$ are increasing in vacancies and decreasing in the number of job-seekers.

Finally, matches have a constant risk of coming to an end, which is expressed by a rate of separation that hits young and older worker matches at the probabilities μ_y and μ_o respectively. Whenever a match is destroyed the job becomes vacant and bears a maintenance cost k, while the worker becomes unemployed[1].

[1]Our framework is very similar to that of Hetze and Ochsen (2006) except for the fact that those authors do not take into account the transition from one age group to another. In their study, the authors also extend a standard model of equilibrium unemployment by the distinction between age specific separation risks and a productivity differential between young and older workers but they are only concerned with the modification of relative group sizes due to the aging process.

Figure 1 summarizes these labor flows and transition probabilities where e_y and e_o stand for a young or an older employed worker respectively.

Figure 1: labor market flows

$$\begin{tikzcd}
e_y \arrow[r, "\eta_y"] \arrow[dd, shift left, "\mu_y"] & e_o \arrow[dr, "\eta_o"] \arrow[dd, shift left, "\mu_o"] & \\
 & & retired \\
u_y \arrow[r, "\eta_y"'] \arrow[uu, shift left, "\theta q(\theta)"] & u_o \arrow[ur, "\eta_o"'] \arrow[uu, shift left, "\theta q(\theta)"] &
\end{tikzcd}$$

2.2 Agents payoffs

Workers and firms are risk neutral and discount the future at rate r. Firms post vacancies that are filled with the endogenous probability $q(\theta)$. We also assume that firms cannot discriminate exante between young and older workers, due to existing legislation. The asset value for an unfilled vacancy (J_v) is then:

$$rJ_v = -k + q(\theta)(J - J_v) \tag{3}$$

where J is the value of a filled job.

However once a firm meets a worker, it can observe his or her age. Then the value of a filled job is different for a young (J_y) or an older worker (J_o)[2]:

$$rJ_y = \gamma - w_y + \mu_y(J_v - J_y) + \eta_y(J_o - J_y) \tag{4}$$

$$rJ_o = \delta\gamma - w_o + (\mu_o + \eta_o)(J_v - J_o) \tag{5}$$

where w_y and w_o denote the wage for young and older workers respectively. The

[2]Without lack of generality, the selling price of the output is assumed to be 1.

expression for the value of a filled job is given by:

$$J \equiv p\pi J_y + (1 - p\pi) J_o \tag{6}$$

The expected income stream of a vacancy is equal to the probability that the vacancy meets a particular worker times the expected rents of a match with this worker. Given $q_y(\theta)$ and $q_o(\theta)$, this depends not only on the stock of unemployed workers but also on the composition of unemployment. Furthermore, when the worker filling the vacancy is young, the firm knows that the worker will become old with probability η_y. Similarly, when the worker is older, the asset value of the filled vacancy takes into account the fact that the worker will go onto retirement with probability η_o.

Workers are heterogeneous with respect to their productivity and their rate of separation. The employment of a younger or an older worker provides different returns to the firms and consequently a job provides a different income for the two groups of workers. We assume that the value of an agent being retired, W_r, is 0 and the values of being employed (W_i) and unemployed (U_i) are given by:

$$rW_y = w_y + \mu_y(U_y - W_y) + \eta_y(W_o - W_y) \tag{7}$$

$$rW_o = w_o + \mu_o(U_o - W_o) + \eta_o(W_r - W_o) \tag{8}$$

$$rU_y = b + \theta q(\theta)(W_y - U_y) + \eta_y(U_o - U_y) \tag{9}$$

$$rU_o = b + \theta q(\theta)(W_o - U_o) + \eta_o(W_r - U_o) \tag{10}$$

where b is the return during job search. Typically, b is composed of unemployment benefits. Firms are assumed to post vacancies until all the rents from new job are exhausted, $J_v = 0$. Then, from (3), (4) and (5), it follows that (see appendix):

$$\frac{k}{q(\theta)} = p\pi \frac{\gamma - w_y + \eta_y \frac{\delta\gamma - w_o}{r+\mu_o+\eta_o}}{r + \mu_y + \eta_y} + (1 - p\pi) \frac{\delta\gamma - w_o}{r + \mu_o + \eta_o} \tag{11}$$

This relationship gives the job creation curve (JC) and gives the combined labor demand of both types of worker by the firm. This condition states that in equilibrium, the expected income from a filled vacancy must equal the total cost of posting it.

2.3 Wage setting

Our analysis considers the opportunity for older workers to go onto retirement. Such an opportunity raises questions about wage formation. In the literature, it is common practice to let wages be determined by an axiomatic Nash bargaining solution. However, studies regarding "on-the-job-search" showed that workers may use outside opportunities to obtain a higher wage in the same job (Burdett and Mortensen, 1998). Recently, Cahuc et al. (2006) considered a model in which employers are allowed to match the offer of a rival employer and showed that workers can exploit the outside offers to obtain a pay rise.

In our context, this argument suggests that older workers use the threat of retiring as a bargaining tool to obtain higher wages. We decided not to follow this route because it is very unlikely that workers renegotiate wages on the basis of retirement opportunities. The vast literature on retirement decision-making has shown that the generosity of social security benefits and positively valued leisure make continued work unattractive (Gruber and Wise, 1999). Among other factors, the lowering of the standard retirement age has contributed to the increase in the so-called "implicit tax" on work. This is what we want to analyze through the impact of η_o.

Wages are the outcome of bilateral Nash bargaining between firms and workers. For each match, the Nash bargaining solution is the w_i that maximizes the weighted match surplus:

$$(W_i - U_i)^{\beta}(J_i - J_v)^{1-\beta} \tag{12}$$

where β gives the bargaining power of workers. The wages solving this problem are (see appendix):

$$w_y = \frac{(1-\beta)b + \beta\gamma\left(1 + \frac{\theta q(\theta)}{r+\mu_y+\eta_y}\right) + \eta_y\beta\frac{\theta q(\theta)}{r+\mu_y+\eta_y}\frac{\delta\gamma - w_o}{r+\mu_o+\eta_o}}{1 + \frac{\theta q(\theta)\beta}{r+\mu_y+\eta_y}} \tag{13}$$

$$w_o = \frac{(1-\beta)b + \beta\delta\gamma\left(1 + \frac{\theta q(\theta)}{r+\mu_o+\eta_o}\right)}{1 + \frac{\theta q(\theta)\beta}{r+\mu_o+\eta_o}} \tag{14}$$

It is important to note that the wages of older and younger workers will diverge according to several factors. Except for extreme values of β, there is a twofold effect of the probability of finding a job. On the one hand, the wage increases with $\theta q(\theta)$ because, if it is easier to find another job, workers accommodate easily to being rejected in the job market and require better wages. On the other hand, the wage decreases with $\theta q(\theta)$ because firms have to wait for a longer period to fill a vacancy and this reduces the total revenue to be shared from the match. But wages differ with age because productivity and separation rates differ with age. Equation (13) also shows that the wage of the young workers will be influenced by their probability of staying with a firm and going into the older workers' group. If they have a positive probability of aging ($\eta_y > 0$), they capture a part of the outcome when old, J_o. This additional term obviously depends on their bargaining power.

2.4 Supply of vacancies

We will now derive the relationship between the supply of vacancies by firms and the key parameters of our framework. Consider a standard matching function: $m = v^{\epsilon}u^{1-\epsilon}$. Since firms post vacancies up to the point where the expected value of a filled job is equal to the total cost of filling it, i.e. $J = k/q(\theta)$, we easily obtain an equation determining θ as a function of the probability of meeting a young or an older workers:

$$\theta = \left(\frac{pu_y J_y + (1-p)u_o J_o}{ku} \right)^{\frac{1}{1-\epsilon}} \tag{15}$$

The supply of vacancies depends on the composition of unemployment. Even if at the moment they open up a vacancy, employers do not know whether they will meet a young or an older worker, they do know the aggregate composition of unemployment, and therefore they can anticipate the probability of meeting each of the worker types. There is no direct analytical solution for θ but it appears that δ plays a crucial role.

Employers may prefer to hire a young or an older worker because the age group has a higher productivity-wage ratio or a lower separation rate. However, firms also know that if they reject the candidature of a worker, it causes further search costs. Firms have to pay k for an additional $1/q(\theta)$ period. Therefore, firms accept applicants, whatever the age group, if the cost of rejection is equal to the gain of employing a better worker. In other words, the age-specific differences have no effect on the value of a job if the free entry condition holds. The value of δ for which this holds, δ^*, is derived from:

$$\delta^* = 1 + \frac{(\mu_o - \mu_y + \eta_o)(1 - \frac{b}{\gamma})}{r + \mu_y + \beta\theta q(\theta)} \tag{16}$$

The productivity level for a job occupied by an older worker being as profitable as a job occupied by a young worker is directly affected by the expected duration of the job. If the exogenous rates of termination of older workers (μ_o, η_o) increase, the required level of productivity increases. In particular, the higher the probability of retirement for an older worker (η_o increases), the higher the level of productivity must be to compensate employers for the shorter expected match duration.

When $\delta < \delta^*$, it is more gainful to employ a younger than an older worker because an older worker does not produce enough to compensate for their higher probability of quitting ($\eta_o > \eta_y$). By contrast, if older workers produce sufficiently more than younger workers, firms may prefer hiring older workers. Therefore, competition between both types of worker induces a standard negative congestion externality, but older workers also create a positive search externality on young workers. This positive externality is similar to that discussed by Gautier (2002) and by Ortega (2000) when considering the productivity differential of skilled and unskilled workers or search costs differential for native and immigrant workers. This result stems from the fact that the more highly productive older workers search for a job, the more attractive it is for employers to open up vacancies. Young workers may benefit from the increase in θ, which makes the probability of finding a job higher. The negative trading externality arises because young and older workers are punished exactly the same for the fact that older workers have a higher probability of quitting, η_o. But when $\delta > \delta^*$, the ratio of vacancies to workers increases in the number of old job-seekers, u_o. In this case, the young workers benefit from the larger supply of vacancies caused by the higher productivity of older workers.

At this point nothing is new, in the present study, in comparison with previous studies. However, when η_o increases, not only does it increase the level of δ^* required to make a job occupied by an older worker as profitable as a job occupied by a young worker but it reduces directly the number of older job-seekers. Indeed, older unemployed workers are concerned with the probability of retirement in a similar way to older employed workers. This, in turn, also has an impact on θ, which modifies the probability of finding a job for all workers. We then obtain a third externality caused by η_o not yet identified.

When older workers go into retirement they affect not only their own labor outcomes but also the whole of the labor market equilibrium. The effect on equilibrium unemployment as well as on the levels of unemployment of younger workers will depend on the value taken by δ. From (15), it can be shown that, except for values of δ higher than δ^{**}, $d\theta/du_o > 0$ [3]. Therefore a value δ^{**} exists for which $d\theta/du_o = 0$ and is negative for any value of $\delta > \delta^{**}$. From this, we can determine how the probability of filling a vacancy will be affected by the variation in u_o:

$$\frac{dq(\theta)}{du_o} = \frac{\partial q(\theta)}{\partial \theta}\frac{d\theta}{du_o} = (\epsilon - 1)\theta^{\epsilon}\frac{d\theta}{du_o} > 0, if \delta < \delta^{**}$$

When $\delta < \delta^{**}$,the probability of retirement, which decreases the number of older unemployed, reduces the market tightness and consequently the probability of finding a job for every worker. Since v is an increasing function of u_o, when u_o decreases because of higher η_o, firms open fewer vacancies. Moreover if $\delta < \delta^{*}$, firms know that older workers are poorly productive. When firms meet an older worker, if they reject the match, they will have to wait for a further period of time before making another match. This increases the cost of the vacancy, which induces them to open up even fewer vacancies. It is only in the case where $\delta > \delta^{**}$ that increased retirement is profitable for both young and older workers.

It is important to note that even though we had assumed that firms open up vacancies to specific age groups, the effect of increased retirement would still depend on the productivity differential. This is because when hiring a young

[3] It can be shown that the sign of $d\theta/du_o$ depends on the value taken by δ. The simulations of next section will clarify this.

worker, firms know that the match will turn into an older match at some point and, as a result, they interiorize the fact that productivity will change.

2.5 Equilibrium unemployment

The model can be closed by writing down the flow conditions in the steady state:

$$\eta_y u_y + \theta q(\theta) u_y = \mu_y (1 - u_y) + \eta_b u_y \tag{17}$$

$$\eta_o u_o + \theta q(\theta) u_o = \mu_o (1 - u_o) + \eta_y u_y \tag{18}$$

where η_b stands for the proportion of young unemployed workers entering the labor market. This corresponds to young people finishing school and entering the labor market as job-seekers. In the steady state, the population growth is zero and there is no aging process of the population. This is a convenient assumption to get rid of any additive effect coming from a change in the size of the cohort[4]. The equilibrium age-specific rates of unemployment are therefore given by:

$$u_y = \frac{\mu_y}{\mu_y + \theta q(\theta) + \eta_y - \eta_b} \tag{19}$$

$$u_o = \frac{\mu_o + \eta_y u_y}{\mu_o + \theta q(\theta) + \eta_o} \tag{20}$$

It is easy to see that, in steady state equilibrium, the unemployment rate for young workers is higher than the unemployment rate for older workers. In the likely case that jobs occupied by young workers are destroyed sooner and according to the unique matching function, the rate at which both types of worker find vacancies is exactly the same, but older workers have a higher probability of quitting their age-specific labor pool ($\eta_o > \eta_y$).

[4]See Shimer (2001) or also Hetze and Ochsen (2006) for an analysis of the effect of a change in the youth cohort's size on the equilibrium unemployment.

The Beveridge curve (BC) is obtained by summing the two unemployment rates weighted at the respective population shares:

$$u = pu_y + (1-p)u_o \tag{21}$$

3 An illustrative simulation

We will carry out some simulations to illustrate the properties of the model and to gain insights into the effect of increased retirement. We divide the population into two age groups corresponding to workers aged 15-54 and workers aged 55-64. This corresponds to the age at which workers may be entitled to retirement opportunities in most OECD countries. Our baseline corresponds to the equilibrium corresponding to the case where there is no productivity difference between workers. Starting from this point, we introduce successively a productivity differential and a positive increase in the probability of retirement to illustrate the effect on equilibrium unemployment.

3.1 Parameter values

The matching function takes the usual Cobb-Douglas form with constant returns to scale. The Cobb-Douglas matching function with constant returns to scale fits well evidence on labor market flows (Blanchard and Diamond, 1989). Unemployment and vacancies are assumed to have equal weights in the matching function (Petrongolo and Pissarides, 2001). The exogenous variables of the model are set at the following values:

$$\begin{array}{llll} \beta = 0.5; & \epsilon = 0.5; & b = 0.3\gamma; & r = 0.05; \\ \mu_y = 0.15; & \mu_o = 0.1; & \gamma = 1; & \delta = 1\ . \end{array}$$

The interest rate is set to 5%. The labor productivity of young workers is, without loss of generality, normalized to 1 and in the baseline case there is no difference in productivity between workers ($\delta = 1$). We set age-specific destruction rates at $\mu_y = 0.15$ and $\mu_o = 0.10$. This to refer to the higher cost of firing an older workers than a young workers [5]. The benefit replacement rate is set to 30% of the productivity level, which is in accordance with the estimations made by Blanchard and Wolfers (2000). The cost of a vacancy is chosen arbitrarily so that the unemployment rates for young and older workers are in the same order of magnitude as in OECD countries.

3.2 Simulations

Table 1 presents the results of the simulations. The baseline values correspond to equilibrium before an increase of η_o. The table shows that the young unemployment rate is 8.5% and the older unemployment rate is 5.3%. Those values are of the same order of magnitude as the unemployment rates of the OECD countries. The total unemployment rate is 8.1%. The critical value of δ^* is 1.04, which means that when older workers produce more than 1.04 times as much as young workers in their jobs, employers actually prefer older workers in those jobs. Consequently the employers open up more vacancies if the ratio of older unemployed workers to young unemployed workers increases.

Whether or not the output of older workers changes, young workers may benefit from older workers looking for a job. When the productivity differential is in favor of young workers, everything else being equal, the young unemployment rate increases by 0.07 of a percentage point, whereas when older workers are highly productive, the young unemployment rate decreases compared to our baseline case. This confirms the role played by δ^* as a pivotal.

[5] See for example the French "Delalande act" which imposes higher severance pay for separation of old workers.

Table 1: Effects of increased retirement: numerical simulations

	U_y^{rate}	U_o^{rate}	U^{rate}	δ^*	δ^{**}
Before increased retirement					
$\delta = 1$(Baseline)	8.45	5.26	8.13	1.04	-
$\delta = 0.8$	8.52	5.29	8.19	1.04	-
$\delta = 1.2$	8.39	5.20	8.07	1.04	-
$\delta = 2.3$	8.04	4.98	7.74	1.04	-
After increased retirement					
$\delta = 1$	9.03	5.00	8.62	1.10	2.20
$\delta = 0.8$	9.09	5.03	8.68	1.11	2.20
$\delta = 1.2$	8.96	4.97	8.52	1.10	2.20
$\delta = 2.3$	7.65	4.52	7.34	1.09	2.20

We then simulate the impact of an increase in the probability of retirement of 10 percentage points by differentiating again the level of productivity of older workers. In this respect, we pay attention to the extreme value of δ, which makes the ratio of vacancies to job-seekers independent of the increase in η_o.

When δ*= 1*, the introduction of early retirement possibilities for older workers increases the unemployment rate of young workers. As stressed previously, an increase in the probability of exit for older workers has a negative impact on labor outcomes. It reduces the value of a filled job for firms, which consequently reduces their supply of vacancies. There are two effects. First, because a job match has a lower duration, its expected value is decreased. Second, given the fact that older workers are going onto retirement earlier and more often than before, the number of unemployed workers decreases, thereby reducing the number of possible applicants for a vacancy. This reduces the probability of a firm filling a vacancy.

When δ*= 0.8*, the productivity of older workers is less than that of young workers, the effect is much higher because it is not profitable for firms to fill a job with older workers. Since there is a positive probability of meeting an older worker, this lengthens the time of a good match. It increases the cost of vacancies.

When δ*= 1.2*, the productivity of older workers is sufficiently high ($\delta = 1.2 > \delta^*$). Young workers are penalized to a lesser extent than in the two previous cases. Since older workers compensate for their higher probability of quitting, firms have an incentive to open up vacancies in order to obtain a match with older workers. Young workers benefit from this. However, at this level, productivity is not high enough ($\delta < \delta^{**}$) to also compensate for the drop in old unemployment. Everything else being equal, the young unemployment rate drops by 0.13 of a percentage point to 8.96% in comparison with the situation where productivity is identical for both types of worker but it increases in comparison with the situation before increased retirement possibilities.

Finally, when δ*= 2.3*, the productivity of older workers is high enough to induce firms to open up even more vacancies and this reduces both young and older unemployment levels. This means that the productivity of older workers has to be more than 2.2 times higher than the productivity of younger workers in order for retirement to be beneficial to young and global unemployment. This result has to be mitigated. There are few reasons to expect such a productivity differential. In the empirical literature, different approaches have been used to estimate the age-productivity relationship and although they are somewhat contradictory in terms of estimated profiles of productivity, they have never found such a productivity differential[6].

Interestingly, we see that in every case, the unemployment of the older workers decreased as a consequence of early retirement. This is not surprising since early retirement directly affects the pool of unemployed workers. However, the magnitudes are different and this is also related to the situation of young workers. Indeed, since unemployment of older worker is directly related to that of younger

[6] A good survey of the literature on age and individual productivity can be found in Skirbekk (2003).

workers, the benefits of increased retirement will be diminished by the effects on the young. When the situation is really bad for young workers, the unemployment rate of older workers is also badly affected.

4 Empirical evidence

We will now compare our theoretical predictions with empirical facts. The model predicts different effects of retirement on unemployment according to the assumptions made on the heterogeneity of workers.

The strategy is to link the theoretical model with an econometric model through the estimation of a reduced-form specification of the unemployment rates. We use OECD data from Bassanini and Duval (2006). This database has been widely used to estimate reduced-form unemployment equations and is consistent for analysis of theoretical models of labor market equilibrium. We retain 12 countries[7] and estimate unemployment as a panel model. The period goes from 1982 to 2003. Since institutions typically have low variation and are highly correlated within a country[8] we introduce fixed country effects and a time trend is also included. The equations to be estimated are of the form:

$$\begin{aligned} log(u_{it}) &= \beta_o + \beta_2 log(v_{it}) + \beta_2 b_{it} + \beta_3 ud_{it} + \beta_4 ep_{it} \\ &\quad + \beta_5 \gamma_{it} + \beta_6 lc_{it} + \beta_7 rr_{it} + \beta_8 TFP_{it} + \beta_9 p_{it} \\ &\quad + \beta_{10} er_{it} + \beta_{11} trend_t + \phi_i + \epsilon_{it} \end{aligned} \tag{22}$$

This equation is very close to the Beveridge curve of theoretical models. The share of the young workers population (p) is given by the observed share of workers

[7]Australia, Canada, Finland, France, Germany, The Netherlands, Norway, Portugal, Spain, Sweden, United Kingdom and United States.

[8]see Blanchard and Wolfers (2000) and Nickell, Nunziata, and Ochel (2005).

aged between 15 and 54 years old in the total active population. The separation rates are made up of a common idiosyncratic shock, which will be approximated by two variables as well as an age-specific hazard rate. First, the total factor productivity shock (TFP) should induce a temporary decline in unemployment. Second, the real interest rate shock (rr) affects negatively capital accumulation and labor productivity, thereby reducing labor demand. We assume that the age-specific separation probability is taken into account through p. We proxy γ not only by the real labor productivity but also by labor costs in real productivity units (lc). We consider the bargaining power of workers approximated by union density (ud).

The matching efficiency, which is taken as exogenous in the theoretical model, is a function of the benefit replacement rate (b) and employment protection (ep) in the econometric model. A change in the replacement rate affects the search behavior of job-seekers in two ways. First, a higher replacement rate makes the search less costly and extends the search for a better match. Consequently, the resulting higher match quality decreases the consequent quit rate. This outweighs the increased search duration and therefore the matching efficiency increases. Second, the search duration can outweigh the quit rate and, as a result, the matching efficiency decreases. Similarly, employment protection may affect the search behavior of firms in two ways. First, since the cost of dismissal increases, firms intend to reduce dismissals and select job candidates cautiously. The matching efficiency increases if the effect of the resulting lower separation rate exceeds the effect of an extended duration of search. The vacancy rate and the replacement rate are directly observable. Our variables are also important in accounting for institutional effects on job destruction such as union density and employment protection, which reduce the flexibility of firms to react to changes in the economic environment.

Thus p controls for unobserved heterogeneity resulting from a change in the age composition but also for other sources of heterogeneity between age groups. However, p does not have to capture effects from a change in the participation of older workers, for a given age structure; this has to be the result of the early retirement opportunity. This is taken into account through an indicator of the replacement rate of early retirement schemes (er). This variable was calculated by Duval (2004) for a number of OECD countries. It has the advantage of avoiding some of the endogeneity problems related to the use of labor force participation or the employment rates of older workers. If the replacement rate of social security benefits increases, it induces more early retirement. Preliminary analysis showed the negative relationship between this variable and the labor force participation and the employment rates of older workers.

The effect of interest in our case is given by β_{10} which shows how a modification of retirement of older workers, changes the unemployment rate, all things being equal. The results are presented in Table 2.

The vacancy rate has the expected effects on the unemployment rate. An increase in the vacancy rate reduces the unemployment rate as unemployed workers find a job more easily. Union density has a positive effect on unemployment. Indeed, there is some evidence in the literature that trade union power in wage setting has a significant impact on unemployment (Nickell and Layard, 1999). Economic slowdown also increases unemployment. The effects of the two considered shocks are significant. This result points to the importance of a cyclical unemployment pattern, which can be explained by macroeconomic shocks. The level of productivity displays a positive effect on unemployment but the labor costs in productivity units have no significant effect; that is, on average, wages increase moderately compared to productivity.

The generosity of the social security system towards early retirement displays a positive and significant effect on unemployment rates. This shows that the

Table 2: Estimation of unemployment equations

	$log(u_{it})$		$log(u_{it}^y)$	
$log(v_{it})$	-0.411***	(-10.03)	-0.446***	(-9.64)
b_{it}	-0.003	(-0.06)	0.008	(1.35)
ud_{it}	0.021***	(5.60)	0.022***	(5.09)
ep_{it}	-0.388***	(-5.16)	-0.445***	(-5.24)
rr_{it}	0.019**	(2.06)	0.021**	(2.01)
TFP_{it}	-1.277**	(-1.96)	-0.872	(-1.19)
γ_{it}	0.056***	(9.87)	0.054***	(8.46)
lc_{it}	-0.011	(-3.65)	-0.014	(-3.99)
p_{it}	3.477**	(2.01)	5.670***	(2.91)
er_{it}	2.103***	(-3.55)	3.027***	(-4.54)
Fixed effects	YES		YES	
Time trend	YES		YES	
Observations	216		216	

Note: t-statistics are in parentheses. ***, **, * stand for significant at the 1%, 5% and 10 % level.

higher probability of early retirement, which results in a decrease in the labor force participation of older workers has a negative impact on the labor outcomes of all workers. Unemployment increases with increased early retirement. The results show that increased retirement, which translates as a reduction in the participation rate of older workers, has a negative impact on the labor market. Because it decreases the length of a match with older workers, increased retirement incites firms to open up fewer vacancies. It therefore has a negative effect on the unemployment rate, which increases. Within this framework, we are not able to identify positive or negative externalities but we observe the overall effect of an increase in retirement opportunities on labor market outcomes, which appears overall to be negative.

We also estimate an age-specific unemployment equation. The last columns of Table 2 shows the interaction with the younger age group. In this case we estimate a ”partial” BC, which relates the youth unemployment rate to its determinants. The results are of the same order of magnitude as for the total unemployment rate. Increased retirement has a negative impact on youth unemployment.

5 Conclusion

In this paper, we have examined the relationship between retirement and unemployment by means of an equilibrium unemployment model with workers of different ages. In the theoretical framework, the productivity of older workers turns out to be a key variable of the labor market. We have shown that in the presence of greater retirement opportunities, young workers may benefit from the high quit rate of older workers if older workers demonstrate sufficiently high productivity compared to younger ones. This means that when older workers are induced to quit the labor market early, older workers need to be very attractive for firms in order to incite firms to keep opening up vacancies.

We performed two kinds of estimation of the model's prediction. First, we simulated the model in taking conventional parameter values that allow a replication of actual OECD labor markets regularities. The results show that the younger unemployment rate is rather badly affected by retirement for the acceptable value of productivity differential. Second, we performed a regression analysis of the Beveridge curve, which is the unemployment rate. This was carried out using data from a panel of OECD countries. The results highlight that the probability of retirement of older workers has a negative impact on the whole population as well as on the unemployment rates of the younger population.

These results are of great interest when one considers the unemployment-reducing mechanism behind the setting of most early retirement programs. The model's predictions and the empirical estimations contradict the lump of labor argument, which assumes that early retirement will decrease unemployment. However, it is important to stress the partial feature of this analysis. In a general equilibrium framework, we could expect some additional effects to emerge from the consumption behavior of individuals. Furthermore, it is also important to question the means of financing early retirement programs. Since these programs are generally based on a pay-as-you-go system, early retirement will raise taxes on labor and hence will raise the wage costs to the firm with the consequence of depressing labor demand.

References

Acemoglu, D. (1997): "Technology, Unemployment and Efficiency," *European Economic Review*, 41, 525–533.

Bassanini, A., and R. Duval (2006): "Employment Patterns in OECD Countries: Reassessing the Role of Policies and Institutions," OECD Social, Employment and Migration Working Paper, No. 35.

Blanchard, O., and P. Diamond (1989): "The Beveridge Curve," *Brookings Papers on Economic Activity*, 2.

Blanchard, O., and J. Wolfers (2000): "The role of shocks and institutions in the rise of European unemployment: the aggregate evidence," *The Economic Journal*, 110, 1–33.

Boldrin, M., J. Dolado, J. Jimeno, and F. Peracchi (1999): "The Future of Pension in Europe," *Economic Policy*, 29, 289–322.

Burdett, K., and D. D. Mortensen (1998): "Wage Differentials, Employer Size and Unemployment," *International Economic Review*, (39), 257–273.

Cahuc, P., F. Postel-Vinay, and J. Robin (2006): "Wage Bargaining with On-the-Job search: Theory and Evidence," *Econometrica*, 74, 323–364.

Diamond, P. (1982): "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies*, 49, 217–227.

Gautier, P. (2002): "Unemployment and Search Externalities in a Model with Heterogenous Jobs and Workers," *Economica*, 69, 21–40.

Gruber, J., and D. Wise (1999): *Social Security and Retirement Around the World*, NBER Volume. University of Chicago Press, Chicago.

Hairault, J.-O., F. Langot, and S. T. (2006): "The interaction between retirement and job search: a global approach to older workers employment," *IZA Working Paper*, (1984).

Hammermesh, D. (1987): "The Demand for Labor in the Long Run," in *Handbook of Labor Economics, Vol. 1*, ed. by O. Ashenfelter, and R. Layard. Elsevier, Amsterdam, North Holland.

Hammermesh, D., and J. Grant (1979): "Econometric Studies of Labor-Labor Substitution and their Implications for Policy," *Journal of Human Resources*, 14, 518–542.

Hebbink, G. (1993): "Production Factors Substitution and Employment by Age group," *Economic Modeling*, 10, 217–224.

Hetze, P., and C. Ochsen (2006): "Age Effects on Equilibrium Unemployment," *Rostock Center Discussion Paper*, (1).

Kapteyn, A., A. Kalwij, and A. Zaidi (2004): "The Myth of Worksharing," *Labour Economics*, (11), 293–313.

Nickell, S., L. Nunziata, and W. Ochel (2005): "Unemployment in the OECD since the 1960s. What do we Know?," *The Economic Journal*, 115, 1–27.

Nickell, S. J., and R. Layard (1999): "Labour market institutions and economic performance," in *Handbook of Labor Economics, Vol. 1*, ed. by O. Ashenfelter, and D. Card. Elsevier, Amsterdam, North Holland.

Ortega, J. (2000): "Pareto-improving Immigration in an Economy with Equilibrium Unemployment," *The Economic Journal*, 110, 92–112.

Petrongolo, B., and C. Pissarides (2001): "Looking into the Black Box: A Survey of the Matching Function," *The Journal of Economic Literature*, XXXIX, 390–431.

Shimer, R. (2001): "The Impact of Young Workers on the Aggregate Labor Market," *The Quaterly Journal of Economics*, 116, 969–1007.

Skirbekk, V. (2003): "Age and Individual Productivity: A Literature Survey," MPIDR Working Paper, No. 028.

6 Appendix

A. Summary of model's variables

Table 3: Model's variables

Variable	Description
p	Share of young population
η_y	Probability of aging for young workers
η_o	Probability of retirement for old workers
γ	Young worker's productivity
$\delta\gamma$	Old worker's productivity
μ_y	Idiosyncratic shock toward young jobs
μ_o	Idiosyncratic shock toward old jobs
b	Return when unemployed
r	Interest rate
k	Cost of vacancies
π	Ratio of u_y/u
β	Workers bargaining power
θ	Market tightness
$q(\theta)$	Probability of filling a vacancy

B. The job creation curve

From the free entry condition and from (4), we have

$$J_y = \frac{k - q_o(\theta)J_o}{q_y(\theta)} = \frac{\gamma - w_y + \eta_y J_o}{r + \mu + \eta_y} \tag{23}$$

Which gives

$$\frac{k}{q_y(\theta)} = \frac{\gamma - w_y}{r + \mu + \eta_y} + \left(\frac{\eta_y}{r + \mu + \eta_y} + \frac{q_o(\theta)}{q_y(\theta)}\right) J_o \tag{24}$$

From 5), we know that:

$$J_o = \frac{\delta\gamma - w_o}{r + \mu + \eta_o} \tag{25}$$

In substituting (25) into (24), we obtain:

$$\frac{k}{q(\theta)} = \frac{q_y(\theta)}{q(\theta)} \frac{\gamma - w_y + \eta_y \frac{\delta\gamma - w_o}{r + \mu_o + \eta_o}}{r + \mu_y + \eta_y} + \frac{q_o(\theta)}{q(\theta)} \frac{\delta\gamma - w_o}{r + \mu_o + \eta_o}$$

Which can be rewrite as:

$$\frac{k}{q(\theta)} = \frac{q_y(\theta)}{q(\theta)} J_y + \frac{q_o(\theta)}{q(\theta)} J_o$$

C. Wage determination

Nash bargaining requires that the wage is given by

$$W_i - U_i = \beta(J_i - U_i + W_i) \tag{26}$$

Let us start with the young workers. From (4) and (7), we have:

$$J_y = \frac{\gamma - w_y + \eta_y J_o}{r + \mu_y + \eta_y}$$

and

$$W_y = \frac{w_y + \mu_y U_y + \eta_y W_o}{r + \mu_y + \eta_y}$$

By substituting in 26, we have:

$$w_y = \beta\gamma + \beta\eta_y J_o - (1-\beta)\eta_y W_o + (1-\beta)(r+\eta_y)U_y \tag{27}$$

From 26, we have that $W_y - U_y = \frac{\beta}{1-\beta} J_y$ and from (9), we obtain:

$$(r+\mu_y)U_y = b + \theta q(\theta)\frac{\beta}{1-\beta}\frac{\gamma - w_y + \eta_y J_o}{r+\mu_y+\eta_y} + \eta_y U_o \tag{28}$$

And in substituting in 27, we obtain:

$$w_y = \frac{(1-\beta)b + \beta\left(\gamma + \frac{\theta q(\theta)\gamma}{r+\mu_y+\eta_y}\right) + \eta_y \frac{\theta q(\theta)\beta}{r+\mu_y+\eta_y}\frac{\delta\gamma - w_o}{r+\mu_o+\eta_o}}{1 + \frac{\theta q(\theta)\beta}{r+\mu_y+\eta_y}}$$

and in an analogous way, for the older workers, we have:

$$w_o = \frac{(1-\beta)b + \beta\left(\delta\gamma + \frac{\theta q(\theta)\delta\gamma}{r+\mu_o+\eta_o}\right)}{1 + \frac{\theta q(\theta)\beta}{r+\mu_o+\eta_o}}$$

D. Derivation of δ^*

From the equilibrium equation of w_y and w_o, we can rewrite J_y and J_o as:

$$J_y = \frac{(1-\beta)(\gamma - b) + \eta_y J_o}{r + \mu_y + \eta_y + \beta\theta q(\theta)}$$

$$J_o = \frac{(1-\beta)(\delta\gamma - b}{r + \mu_o + \eta_o + \beta\theta q(\theta)}$$

The value of δ^* is simply obtained by equalizing J_y to J_o:

$$\delta^* = 1 + \frac{(\mu_o - \mu_y + \eta_o)(1 - \frac{b}{\gamma})}{r + \mu_y + \beta\theta q(\theta)}$$

E. Supply of vacancies

If we assume a standard functional form for m:

$$m = v^{\epsilon}(u)^{1-\epsilon}$$

From 3, 4 5, we have:

$$rJ_v = \frac{m}{v}\left(\frac{pu_yJ_y + (1-p)u_oJ_o}{u} - J_v\right) - k = 0$$

And by simple arithmetics, we obtain:

$$v = \left(\frac{pu_yJ_y + (1-p)u_oJ_o}{ku^{\epsilon}}\right)^{\frac{1}{1-\epsilon}}$$

We see that the supply of vacancies also depends on the composition of unemployment. Even if at the moment they open a vacancy, employers do not know whether they will meet a young or an old worker, they do know the aggregate composition of unemployment, and therefore they can calculate the probability of meeting each of the worker types.

F. The relationship between θ and u_o

From (15), we obtain:

$$\frac{d\theta}{du_o} = \left(\frac{1}{1-\epsilon}\right)\left(\frac{u_y J_y + u_o J_o}{ku}\right)^{\frac{\epsilon}{1-\epsilon}}\left(\frac{ku_y\left(pJ_o - (1-p)J_y\right)}{\left(k\left(pu_y + (1-p)u_o\right)\right)^2}\right)$$

The first and the second terms of the right-hand side are positive. The third term is positive or negative according to the value taken by δ. The sign of the derivative will be:

1. $\frac{d\theta}{du_o} \geq 0$ if $\delta \leq \delta^* *$
2. δ^{**} is such that $J_o > \frac{1-p}{p} J_y$